Embedded Discontinuity Finite Element Method (ED-FEM) for Modeling Fiber Failures in Random Fiber Networks

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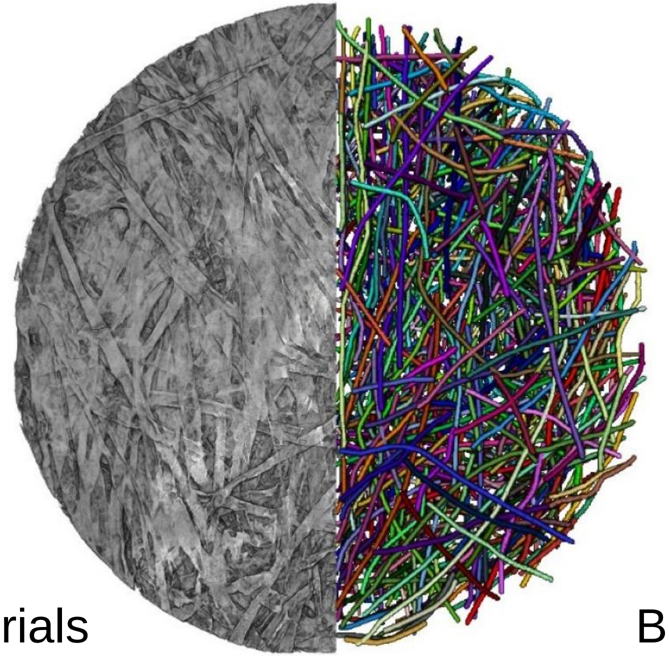
Motivation

Fiber network = fiber-fiber interactions + single fiber segments

Not understood

(Tojaga et al., 2021)

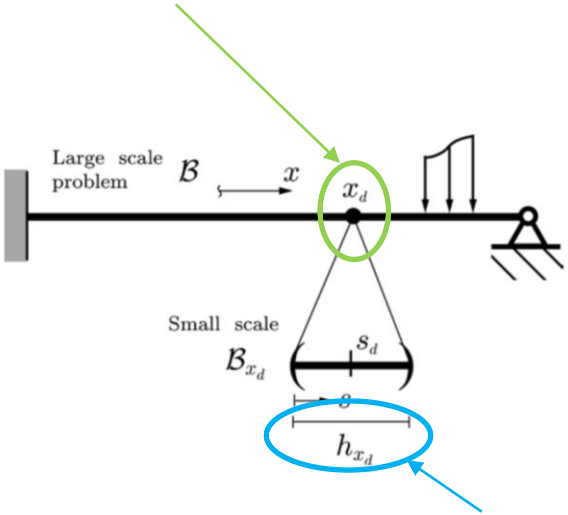


Microstructure of paper materials

Beam network representation

Why use ED-FEM for fracture of beams?

Incorporates a fracture process zone (FPZ) into a continuum



(Ehrlich & Armero, 2005)

Independent of characteristic length scale parameter because the FPZ is a point: $h_{x_d} \to 0$

- Computational efficiency due to one-dimensional continuum nature of beam
- Coupled multi-field problem much like phase field modeling

Theory of strong discontinuities

Heaviside step function: $H(x) = \begin{cases} 0; x > x_{midpoint} \\ 1; x \leq x_{midpoint} \end{cases}$

Enhanced displacement field of beam

$$\boldsymbol{u} = \boldsymbol{u}_s + H(x)\boldsymbol{\xi}$$

Jump at mid-point $x_{midpoint}$ of beam

Generalized displacement field of beam

Enhanced strain field

After some derivation we arrive at:

Dirac delta function: $\delta_{midpoint} = \frac{\partial H(x)}{\partial x}$

Enhanced strain field of beam

$$\boldsymbol{\varepsilon} = \underbrace{\boldsymbol{Bd} + \boldsymbol{G\xi}}_{\text{Bulk material}} + \underbrace{\delta_{midpoint}\boldsymbol{\xi}}_{\text{Discontinuity}}$$

Jump ξ interpolation matrix $\boldsymbol{G} = -\boldsymbol{I}/L_{beam}$

Strain-displacement matrix $\boldsymbol{B}$ and nodal displacement vector $\boldsymbol{d}$

Internal virtual work

$$\delta w_{int} = \int_{L_{beam}} \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma}\, dx$$

Enhanced

$$= \delta \boldsymbol{d}^T \int_{L_{beam}} \boldsymbol{B}^T \boldsymbol{\sigma}\, dx + \delta \boldsymbol{\xi}^T \left[\int_{L_{beam}} \boldsymbol{G\sigma}\, dx + \boldsymbol{\sigma}_{midpoint} \right]$$

Standard FEM

Residual at discontinuity $\bar{\boldsymbol{r}}_e = \boldsymbol{0}$ → Local equilibrium: $\boldsymbol{\sigma}_{midpoint} = \boldsymbol{\sigma}$

Linearization of internal force and local equilibrium

$$\Delta \boldsymbol{f}_{int} = \int_{L_{beam}} \boldsymbol{B}^T \Delta\boldsymbol{\sigma}\, dx$$

$$= \int_{L_{beam}} \boldsymbol{B}^T \boldsymbol{C}\boldsymbol{B}\, dx \Delta\boldsymbol{d} + \int_{L_{beam}} \boldsymbol{B}^T \boldsymbol{C}\boldsymbol{G}\, dx \Delta\boldsymbol{\xi}$$

$$\int_{L_{beam}} \boldsymbol{G}^T \Delta\boldsymbol{\sigma}\, dx + \Delta\boldsymbol{\sigma}_{midpoint}$$

$$= \left(\int_{L_{beam}} \boldsymbol{G}^T \boldsymbol{C}\boldsymbol{B}\, dx + \boldsymbol{C}\boldsymbol{B}\right)\Delta\boldsymbol{d} + \left(\int_{L_{beam}} \boldsymbol{G}^T \boldsymbol{C}\boldsymbol{G}\, dx + \boldsymbol{H}\right)\Delta\boldsymbol{\xi}$$

$$\begin{bmatrix} \boldsymbol{K}_{dd} & \boldsymbol{K}_{d\xi} \\ \boldsymbol{K}_{\xi d} & \boldsymbol{K}_{\xi\xi} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{d} \\ \Delta\boldsymbol{\xi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{r}_e \\ \bar{\boldsymbol{r}}_e = \boldsymbol{0} \end{bmatrix}$$

$$(\boldsymbol{K}_{dd} - \boldsymbol{K}_{\xi d}\boldsymbol{K}_{\xi\xi}^{-1}\boldsymbol{K}_{\xi d})\Delta\boldsymbol{d} = \boldsymbol{r}_e$$

Implementation is non-positive definite

Proposed staggered scheme

$$\boldsymbol{K}_{dd}\Delta\boldsymbol{d} = \boldsymbol{r}_e = \boldsymbol{f}_{ext} - \int_{L_{beam}} \boldsymbol{B}^T\boldsymbol{\sigma}\, dx \quad ; \quad \boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{d}^n, \boldsymbol{\xi}^{n-1})$$

If $\boldsymbol{r}_e = \boldsymbol{0}$, equilibrium, else perform Gauss point computation of $\boldsymbol{\xi}^n$

Constitutive model

Assumption: Elastic unloading in bulk material

Before failure:

Yield criterion: $\Phi^Y = |\sigma| - [\sigma^Y + K\alpha]$

Yield stress resultant σ^Y

Hardening modulus K

Hardening variable α

At failure and beyond:

Bulk material: $\boldsymbol{\sigma} = \boldsymbol{C}[\boldsymbol{Bd} - \boldsymbol{\varepsilon}^p + \boldsymbol{G\xi}]$

Elastic structural stiffness $\boldsymbol{C}$

Strain in bulk material $\boldsymbol{Bd} + \boldsymbol{G\xi}$

Failure criterion: $\Phi^F = |\sigma| - [\sigma^F + H\beta]$

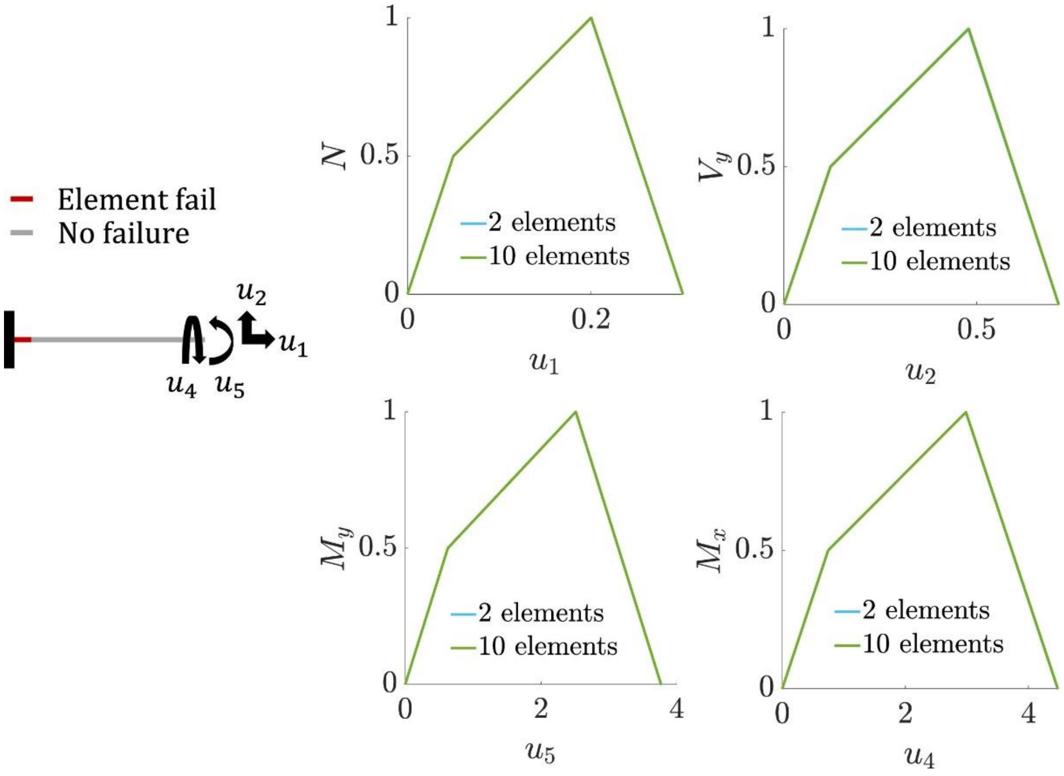
Failure stress resultant σ^F

Softening modulus H

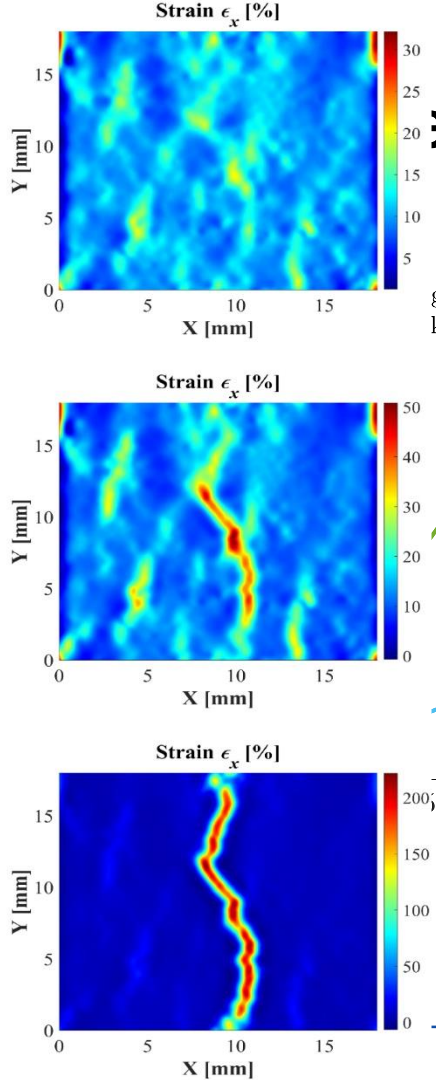
Softening variable β

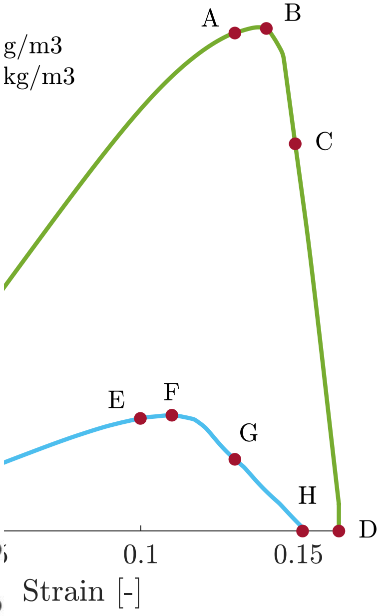
Local equilibrium: $\boldsymbol{\sigma}_{midpoint} = \boldsymbol{\sigma}$

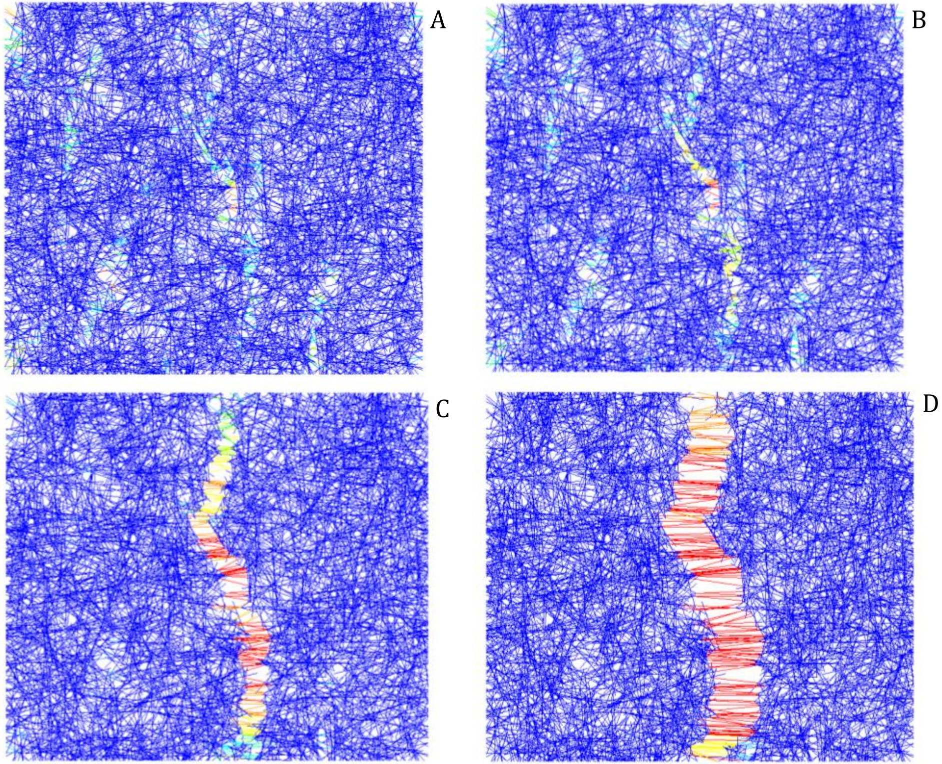
Mesh independence



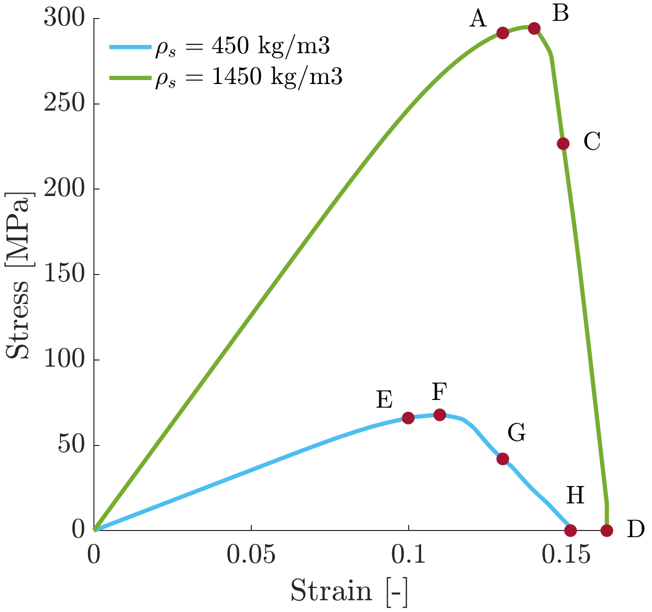
e fiber network

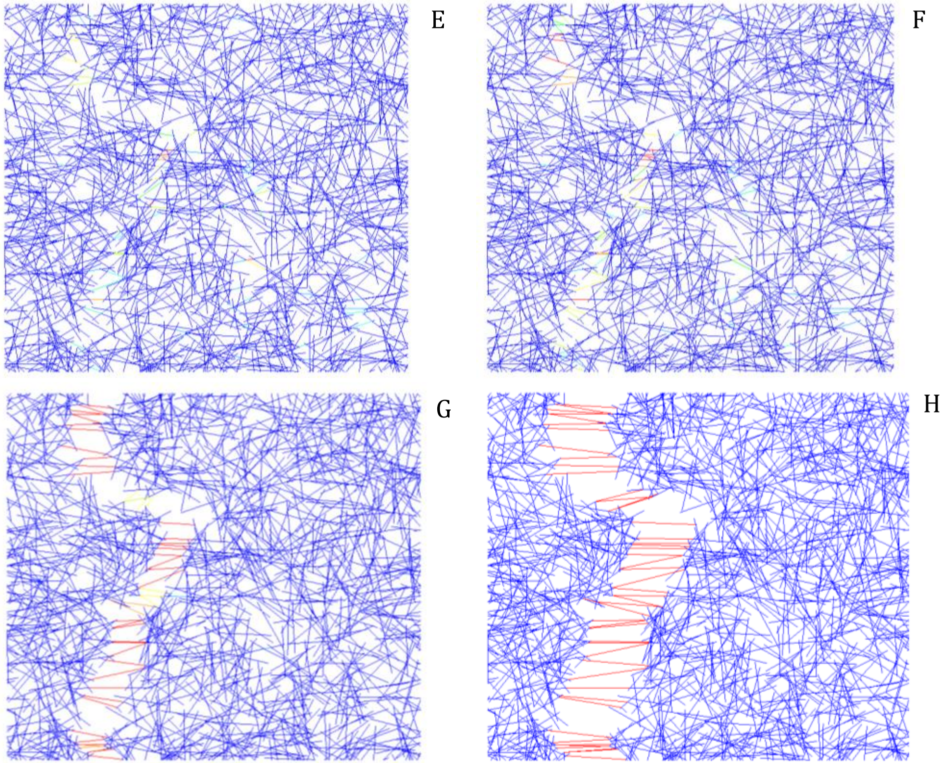






Sparse fiber network





Fiber failures do contribute to the nonlinear stress-strain response of fiber networks!

Summary

1) Robust staggered scheme for ED-FEM applied to fracture of beam structures
2) Implementation in commerical finite element software (ANSYS)
3) FORTRAN source code freely available

Reference:

V. Tojaga, A. Kulachenko, S. Östlund, T.C. Gasser, Modeling multi-fracturing fibers in fiber networks using elastoplastic Timshenko beam finite elements with embedded strong discontinuities – Formulation and staggered algorithm, Comput. Methods Appl. Mech. Eng. (2021).
https://doi.org/10.1016/j.cma.2021.113964