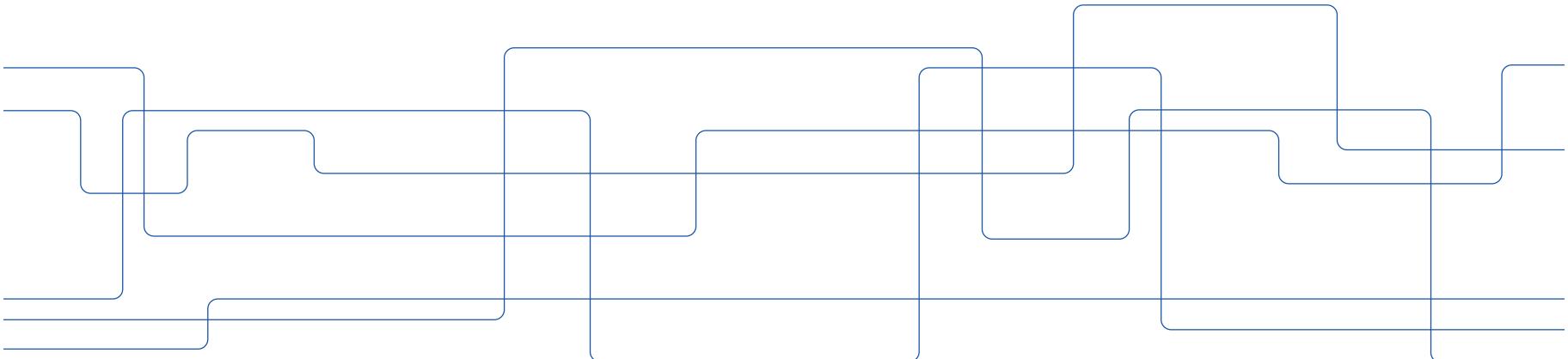




Embedded Discontinuity Finite Element Method (ED-FEM) for Modeling Fiber Failures in Random Fiber Networks

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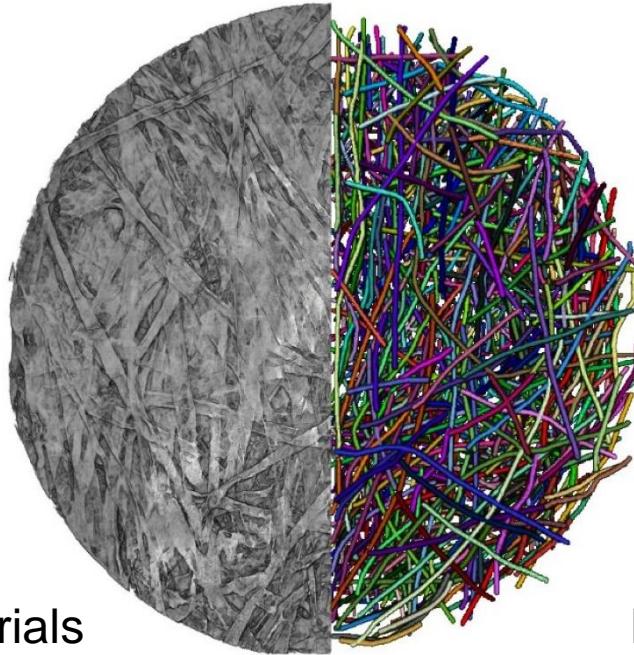
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Motivation

Fiber network = fiber-fiber interactions + single fiber segments

(Tojaga et al., 2021)

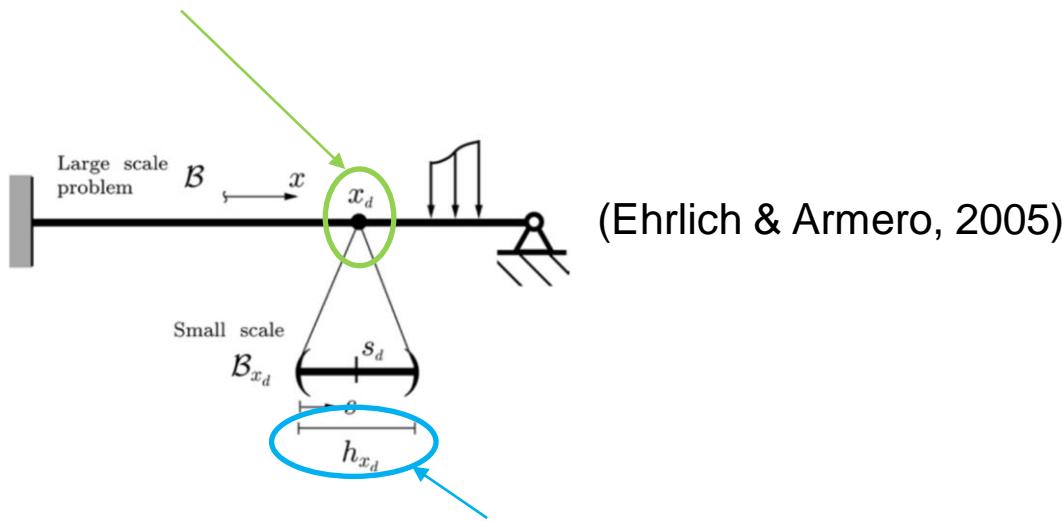


Beam network representation

Not understood

Why use ED-FEM for fracture of beams?

Incorporates a fracture process zone (FPZ) into a continuum



(Ehrlich & Armero, 2005)

Independent of characteristic length scale parameter because the FPZ is a point: $h_{x_d} \rightarrow 0$

- Computational efficiency due to one-dimensional continuum nature of beam
- Coupled multi-field problem much like phase field modeling

Theory of strong discontinuities

$$\text{Heaviside step function: } H(x) = \begin{cases} 0; & x > x_{\text{midpoint}} \\ 1; & x \leq x_{\text{midpoint}} \end{cases}$$

Enhanced displacement field of beam

$$u = u_s + H(x)\xi$$

Jump at mid-point x_{midpoint} of beam

Generalized displacement field of beam

Enhanced strain field

After some derivation we arrive at:

Dirac delta function: $\delta_{midpoint} = \frac{\partial H(x)}{\partial x}$

Enhanced strain field of beam

$$\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{d} + \mathbf{G}\xi + \delta_{midpoint}\xi$$

Diagram illustrating the enhanced strain field of a beam element. The equation shows the strain $\boldsymbol{\varepsilon}$ as the sum of three terms: the bulk material contribution $\mathbf{B}\mathbf{d}$ (indicated by a blue arrow), the discontinuity contribution $\mathbf{G}\xi$ (indicated by an orange arrow), and the midpoint discontinuity contribution $\delta_{midpoint}\xi$ (indicated by a red arrow). A yellow bracket groups the discontinuity terms ($\mathbf{G}\xi + \delta_{midpoint}\xi$). A red arrow points from the term $\delta_{midpoint}\xi$ to the text "Dirac delta function: $\delta_{midpoint} = \frac{\partial H(x)}{\partial x}$ ". A blue arrow points from the term $\mathbf{B}\mathbf{d}$ to the text "Strain-displacement matrix \mathbf{B} and nodal displacement vector \mathbf{d} ". An orange arrow points from the term $\mathbf{G}\xi$ to the text "Jump ξ interpolation matrix $\mathbf{G} = -\mathbf{I}/L_{beam}$ ". Below the equation, the terms are labeled: "Bulk material" under $\mathbf{B}\mathbf{d}$, "Discontinuity" under $\mathbf{G}\xi + \delta_{midpoint}\xi$, and "Jump ξ interpolation matrix" under $\mathbf{G} = -\mathbf{I}/L_{beam}$.

Internal virtual work

$$\begin{aligned}\delta w_{int} &= \int_{L_{beam}} \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dx \\ &= \delta \mathbf{d}^T \int_{L_{beam}} \mathbf{B}^T \boldsymbol{\sigma} dx + \delta \boldsymbol{\xi}^T \left[\int_{L_{beam}} \mathbf{G} \boldsymbol{\sigma} dx + \boldsymbol{\sigma}_{midpoint} \right]\end{aligned}$$

The equation shows the internal virtual work δw_{int} as the sum of two terms. The first term, enclosed in a green oval labeled "Standard FEM", is the standard finite element form: $\delta \mathbf{d}^T \int_{L_{beam}} \mathbf{B}^T \boldsymbol{\sigma} dx$. The second term, enclosed in a blue oval labeled "Enhanced", is the enhanced form: $\delta \boldsymbol{\xi}^T \left[\int_{L_{beam}} \mathbf{G} \boldsymbol{\sigma} dx + \boldsymbol{\sigma}_{midpoint} \right]$. A brace under the enhanced term indicates it is added to the standard term.

Residual at discontinuity $\bar{\mathbf{r}}_e = \mathbf{0} \rightarrow$ Local equilibrium: $\boldsymbol{\sigma}_{midpoint} = \boldsymbol{\sigma}$

Linearization of internal force and local equilibrium

$$\begin{aligned}
 \Delta f_{int} &= \int_{L_{beam}} \mathbf{B}^T \Delta \boldsymbol{\sigma} dx \\
 &= \underbrace{\int_{L_{beam}} \mathbf{B}^T \mathbf{C} \mathbf{B} dx}_{\Delta d} \Delta d + \underbrace{\int_{L_{beam}} \mathbf{B}^T \mathbf{C} \mathbf{G} dx}_{\Delta \xi} \Delta \xi \\
 &\quad \int_{L_{beam}} \mathbf{G}^T \Delta \boldsymbol{\sigma} dx + \Delta \boldsymbol{\sigma}_{midpoint} \\
 &= \left(\int_{L_{beam}} \mathbf{G}^T \mathbf{C} \mathbf{B} dx + \mathbf{C} \mathbf{B} \right) \Delta d + \left(\int_{L_{beam}} \mathbf{G}^T \mathbf{C} \mathbf{G} dx + \mathbf{H} \right) \Delta \xi
 \end{aligned}$$

$$\left[\begin{array}{cc} \mathbf{K}_{dd} & \mathbf{K}_{d\xi} \\ \mathbf{K}_{\xi d} & \mathbf{K}_{\xi\xi} \end{array} \right] \begin{bmatrix} \Delta d \\ \Delta \xi \end{bmatrix} = \begin{bmatrix} \mathbf{r}_e \\ \bar{\mathbf{r}}_e = \mathbf{0} \end{bmatrix}$$

$$(\mathbf{K}_{dd} - \mathbf{K}_{\xi d} \mathbf{K}_{\xi\xi}^{-1} \mathbf{K}_{\xi d}) \Delta d = \mathbf{r}_e$$

Implementation is non-positive definite

Proposed staggered scheme

$$K_{dd} \Delta d = r_e = f_{ext} - \int_{L_{beam}} B^T \sigma dx \quad ; \quad \sigma = \sigma(d^n, \xi^{n-1})$$

If $r_e = 0$, equilibrium, else perform Gauss point computation of ξ^n



Constitutive model

Assumption: Elastic unloading in bulk material

Before failure:

Yield criterion: $\Phi^Y = |\sigma| - [\sigma^Y + K\alpha]$

Yield stress resultant σ^Y

Hardening modulus K

Hardening variable α

At failure and beyond:

Bulk material: $\sigma = C[Bd - \epsilon^p + G\xi]$

Elastic structural stiffness C

Strain in bulk material $Bd + G\xi$

Failure criterion: $\Phi^F = |\sigma| - [\sigma^F + H\beta]$

Failure stress resultant σ^F

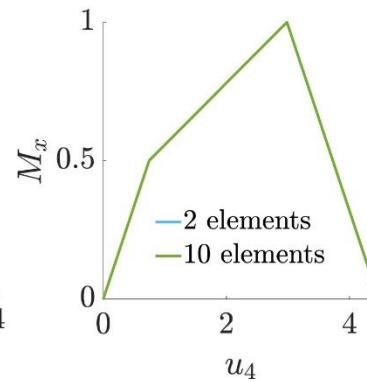
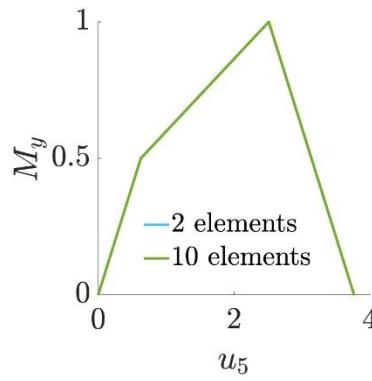
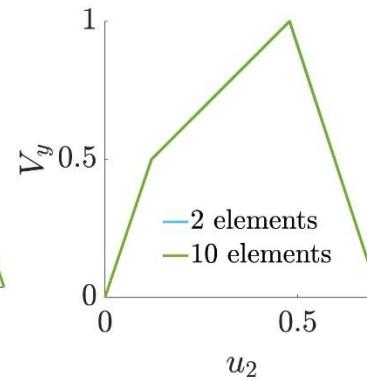
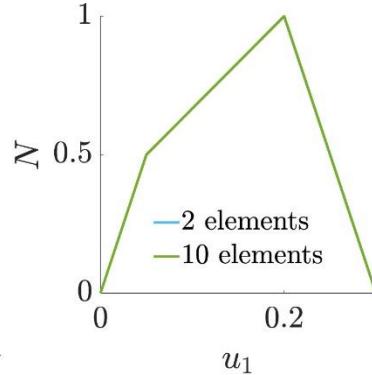
Softening modulus H

Softening variable β

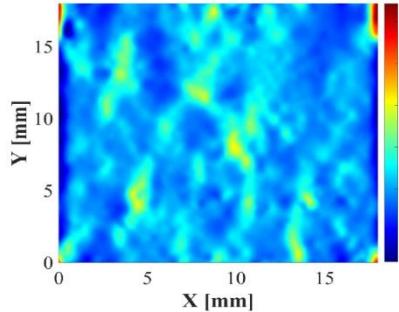
Local equilibrium: $\sigma_{midpoint} = \sigma$

Mesh independence

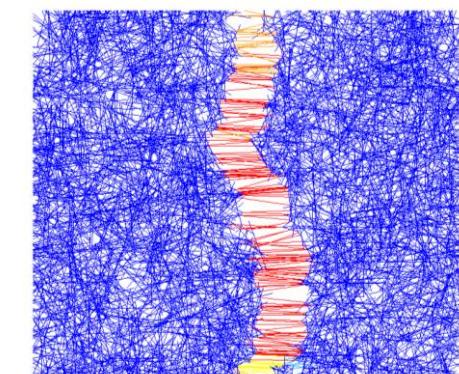
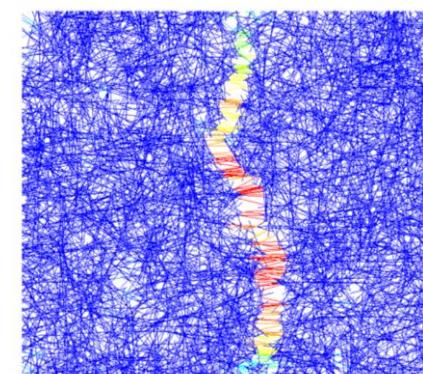
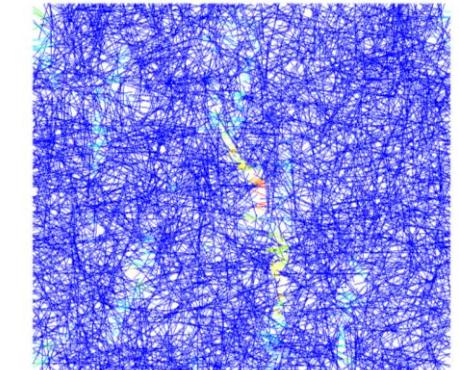
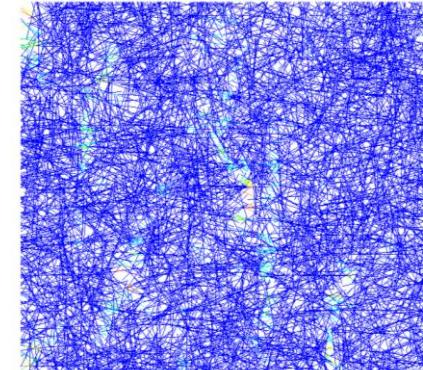
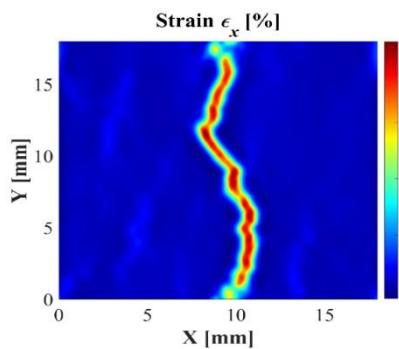
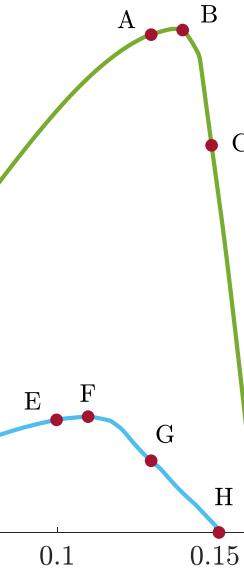
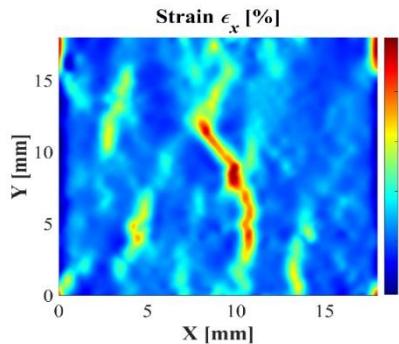
- Element fail
- No failure



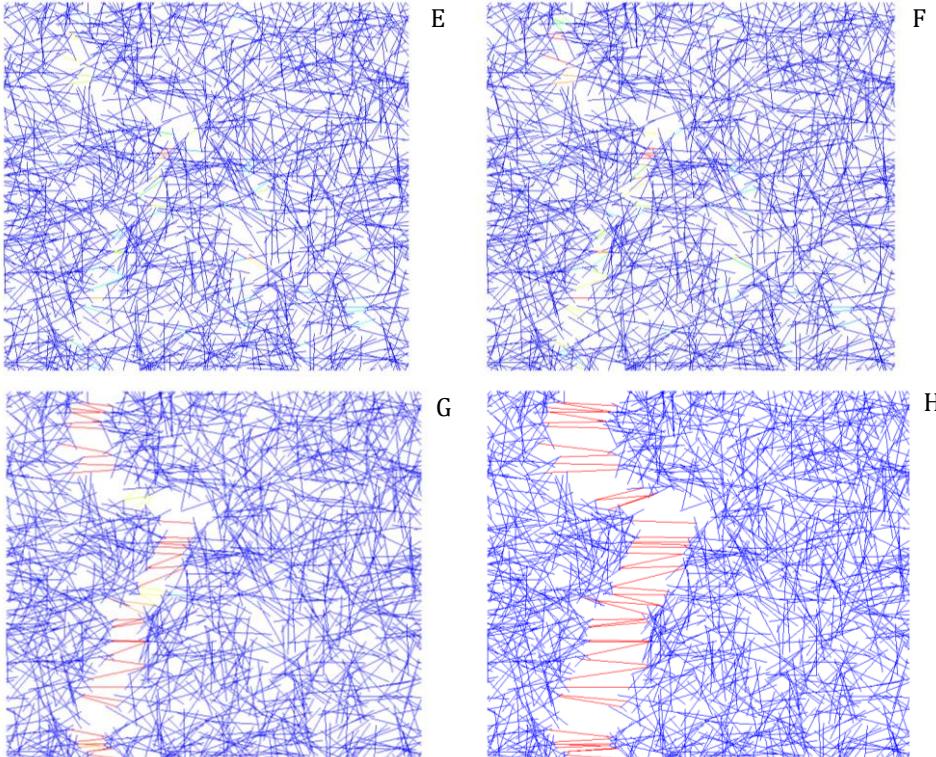
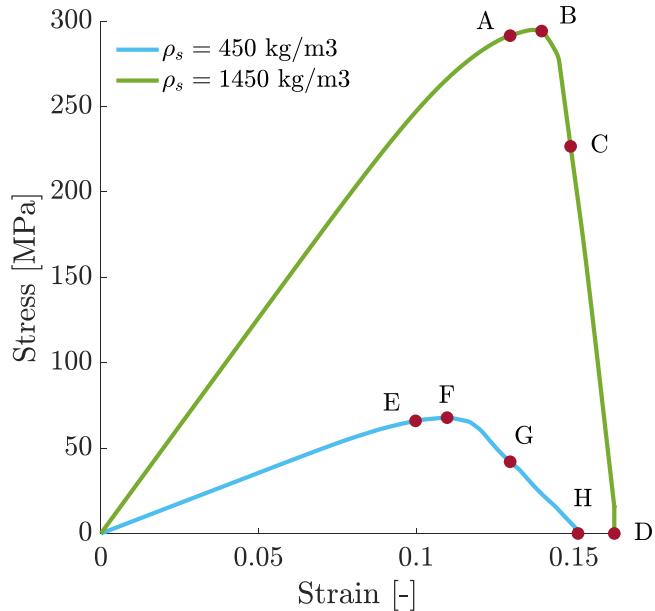
Strain ϵ_x [%]



fiber network



Sparse fiber network



Fiber failures do contribute to the nonlinear stress-strain response of fiber networks!



Summary

- 1) Robust staggered scheme for ED-FEM applied to fracture of beam structures
- 2) Implementation in commercial finite element software (ANSYS)
- 3) FORTRAN source code freely available

Reference:

V. Tojaga, A. Kulachenko, S. Östlund, T.C. Gasser, Modeling multi-fracturing fibers in fiber networks using elastoplastic Timoshenko beam finite elements with embedded strong discontinuities – Formulation and staggered algorithm, Comput. Methods Appl. Mech. Eng. (2021).

<https://doi.org/10.1016/j.cma.2021.113964>